Saxon

Progression in Calculation Appendices

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Magnitude of Calculations

Year 1 – U + U, U + TU (numbers up to 20) including adding zero, U – U, TU – U (numbers up to 20) including subtracting zero, U x U, U ÷ U

Year 2 - TU + U, TU + multiples of 10, TU + TU, U + U + U, TU - U, TU – tens, TU – TU, TU x U, U ÷ U

Year 3 – add numbers with up to three-digits, HTU + multiples of 10, HTU + multiples of 100, subtract numbers up to three-digits, HTU – U, HTU – multiples of 10, HTU – multiples of 100, HTU – HTU, TU x U, TU ÷ U

Year 4 - add and subtract numbers with up to four-digits, ThHTU + ThHTU, ThHTU - ThHTU, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, TU x U, HTU x U, TU x U, multiply by zero and one, TU ÷ U, HTU ÷ U

Year 5 – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, ThHTU x U, ThHTU x TU, HTU x TU, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to the context)

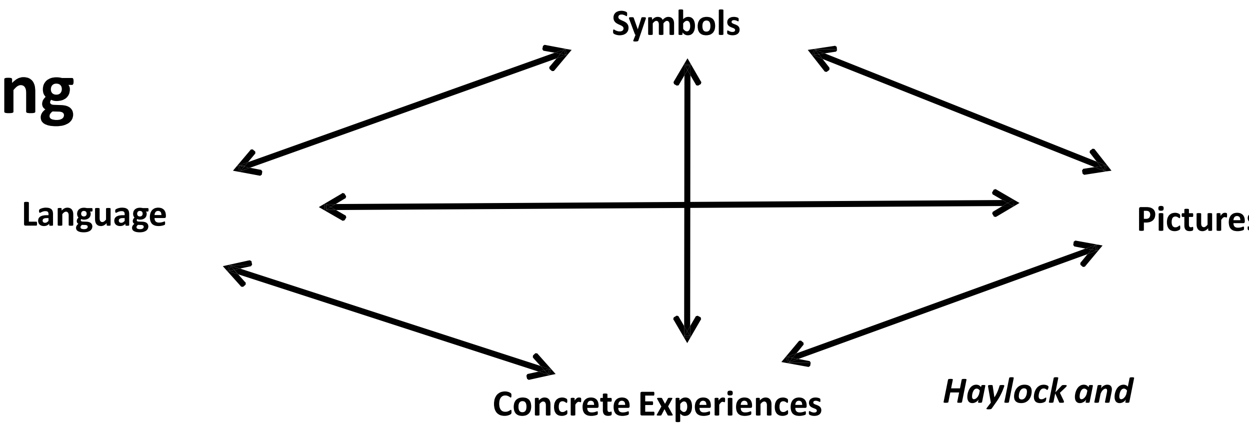
Year 6 - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU, multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. … pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

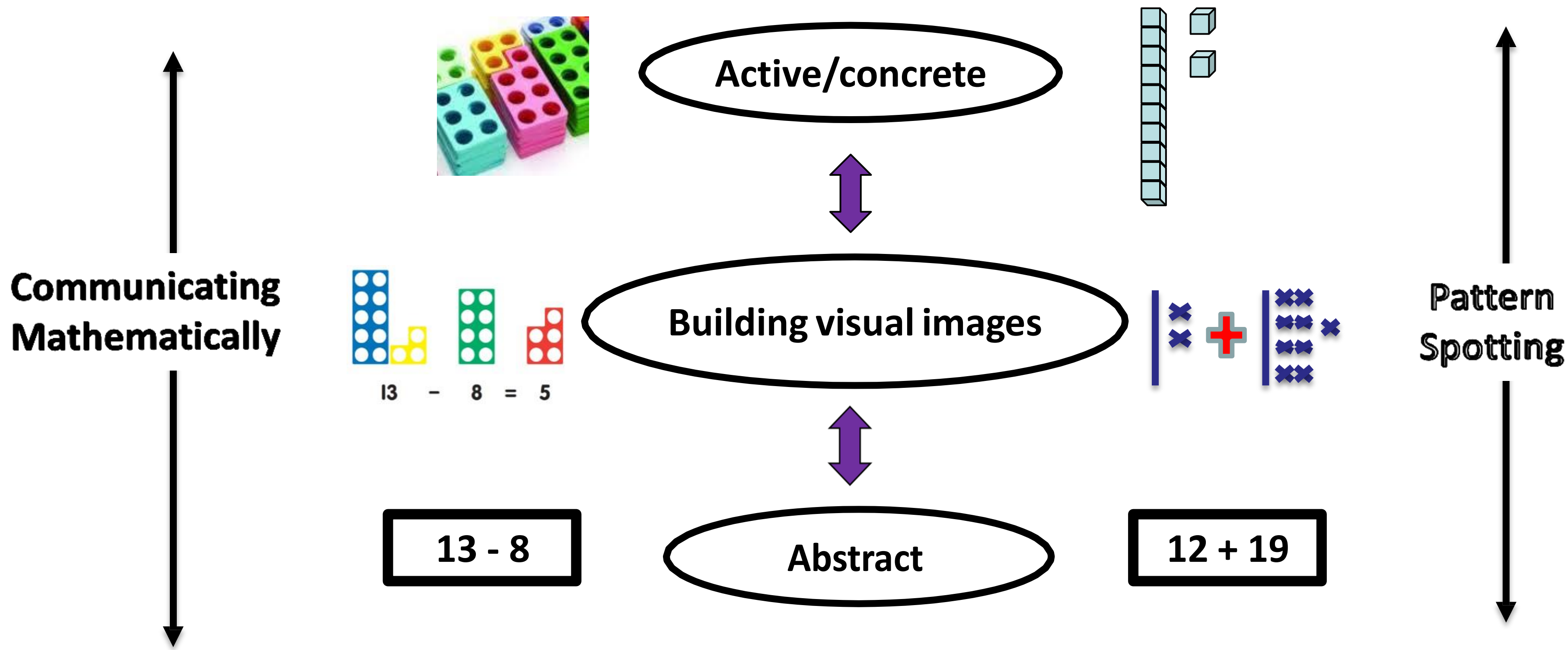
National Curriculum 2014

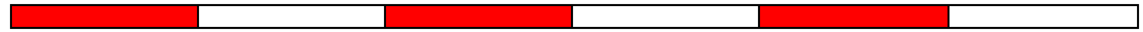
Structuring Learning

Children __must__ have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.



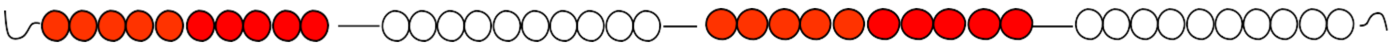
Haylock and Cockburn (2008)



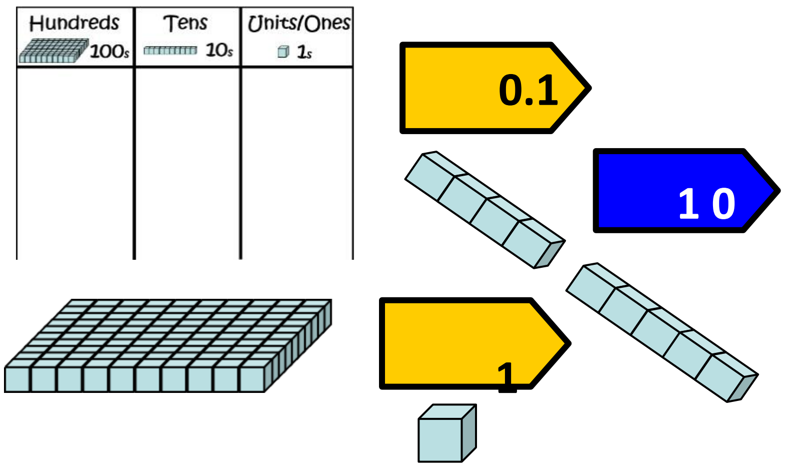


counting stick

bead string

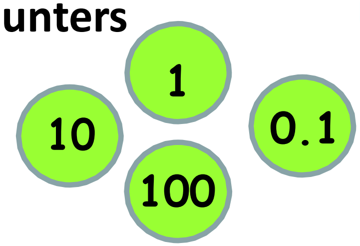


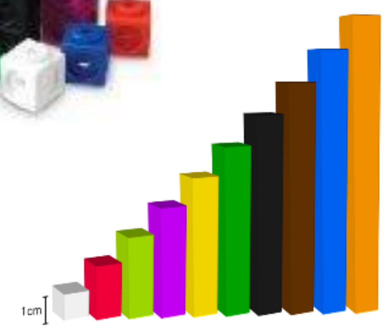
place value apparatus



Multilink

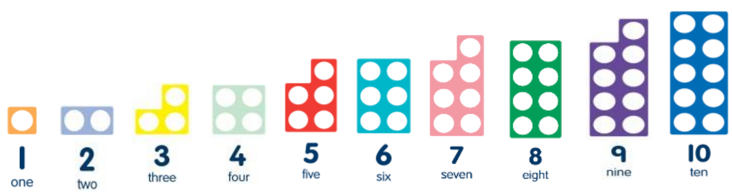
place value counters

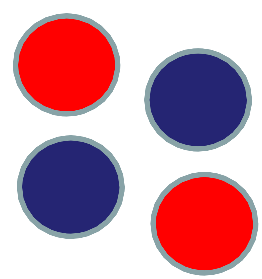




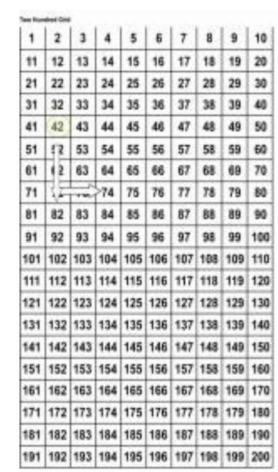
Cuisenaire

Numicon



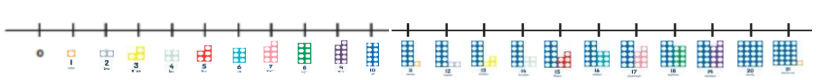


double sided counters



number grids
100 and 200

number line

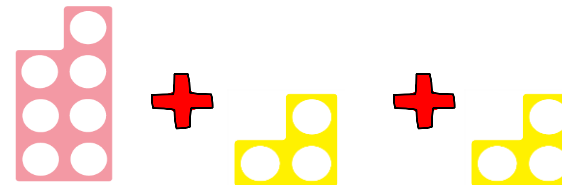


Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

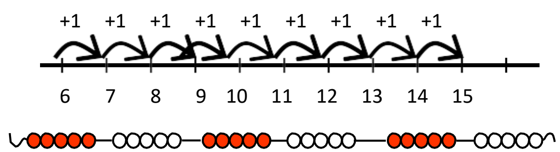
Aggregation

Union of two sets
How many/much altogether?
The total



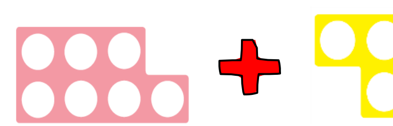
Augmentation

Start at and count on
Increase by
Go up by

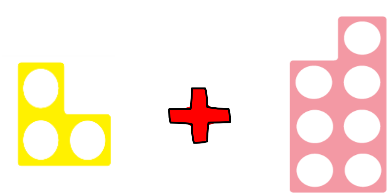


Commutative law

Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)



Is the same as/equal to (=)



addend + addend = sum

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

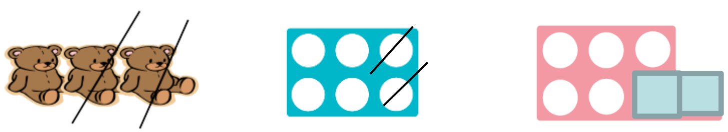
Partitioning

Take away

… how many left?

How many are not?

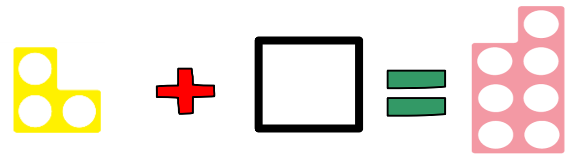
How many do not?

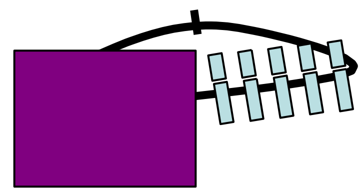


Inverse-of-addition

What must be added?

How many (much) more needed?





There are ten pegs on the hanger – how many are covered?

Comparison

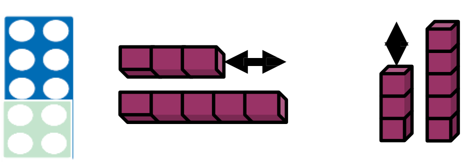
What is the difference?

How many more?

How many less (fewer)?

How much greater?

How much smaller?



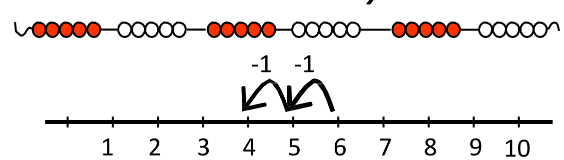
'two more than three is five or two less than five is three'

Reduction

Start at and reduce by

Count back by

Go down by



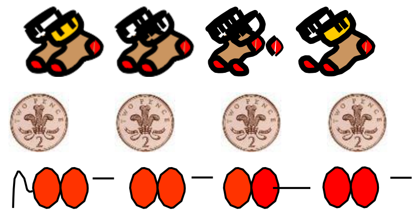
minuend – subtrahend = difference

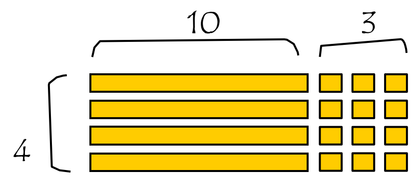
Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

Repeated addition

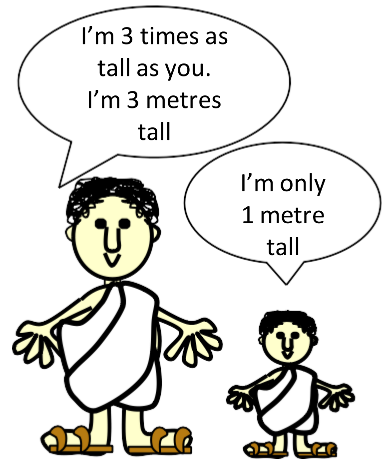
So many lots (sets) of so many
How many (how much) altogether
Per, each

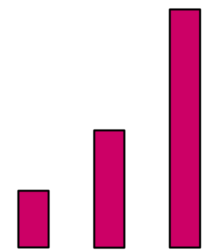




Scaling

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)





scaling with Cuisenaire

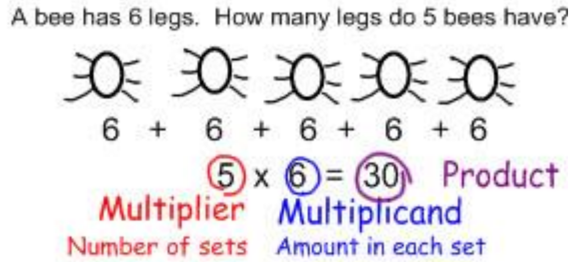
Commutative law

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

a x b and b x a are equal

4 x 2 is the same as/equal to 2 x 4

multiplier x multiplicand = product



Multiplication 4x3= ☐ 4 groups of 3

multiplier x multiplicand=product

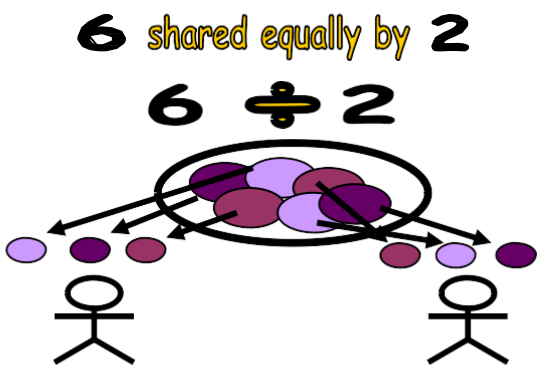
Karen had 4 bags of apples. multiplier
Each bag had 3 apples. multiplicand
How many apples did she have in all? product

Structures for Division (Haylock and Cockburn 2008)

Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

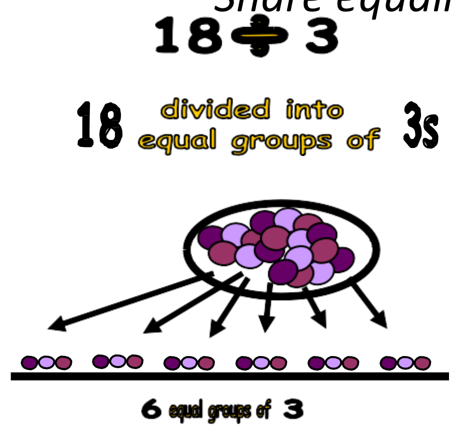
Equal-sharing

Sharing equally between
How many (much) each?

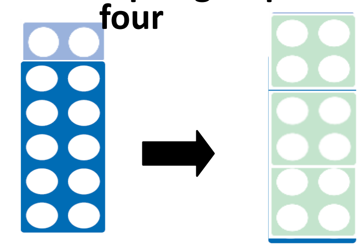


Inverse of multiplication (Grouping)

So many lots (sets/groups) of so many
Share equally in to groups of …



Divide twelve into equal groups of four



= 3

Make 12

Overlay groups of four

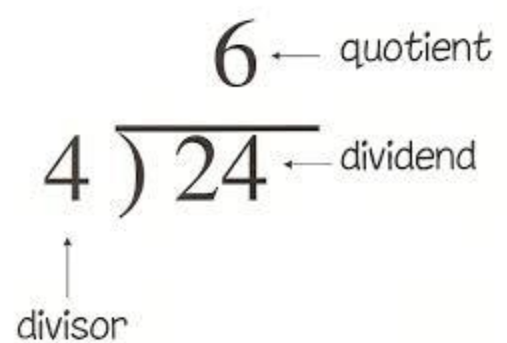
Ratio structure

comparison
inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as long as Ella's. If Jo walks to school in 30 minutes how long does it take Ella?

dividend ÷ divisor = quotient



Equivalence

=

equals

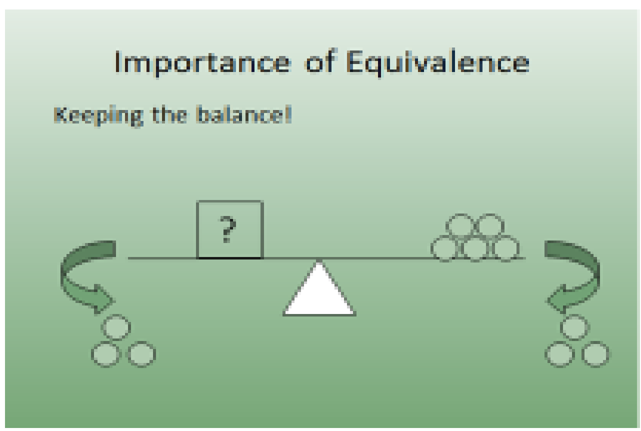
equivalent to

is the same as

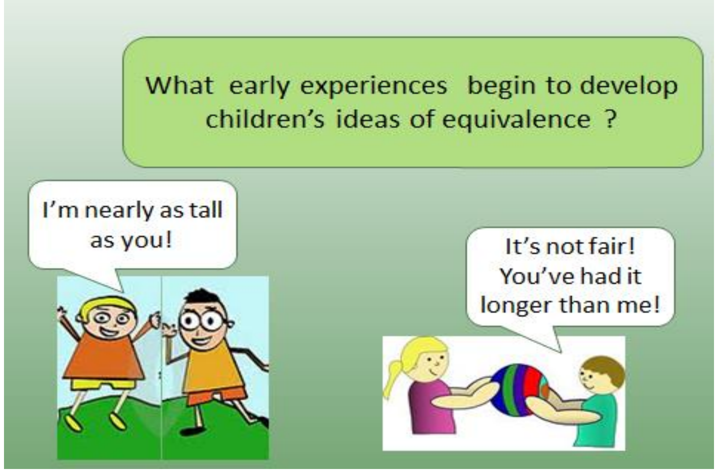
is the same amount as

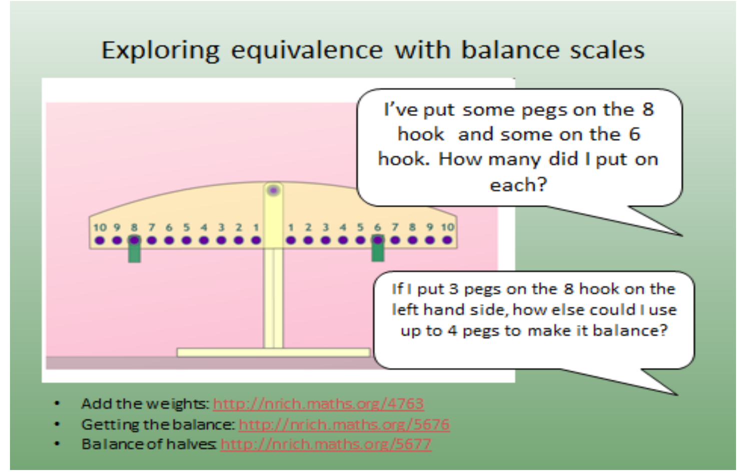
balances

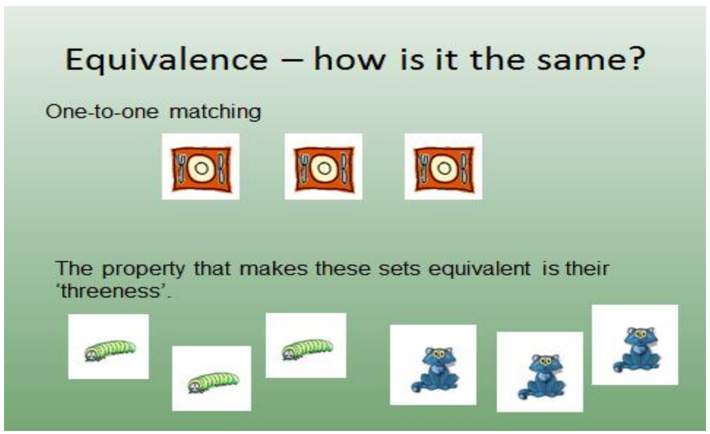
is the same on both sides

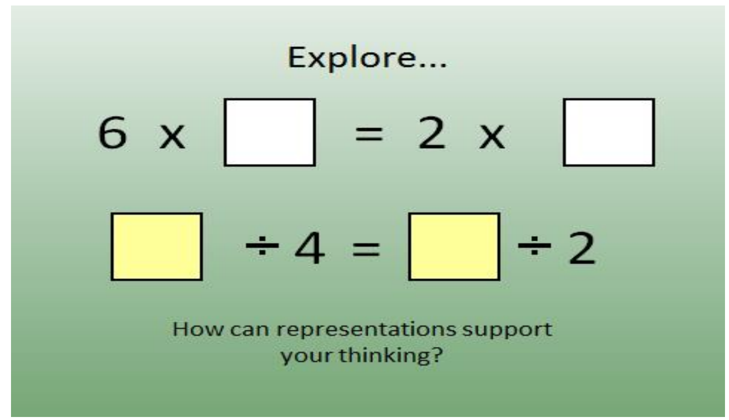


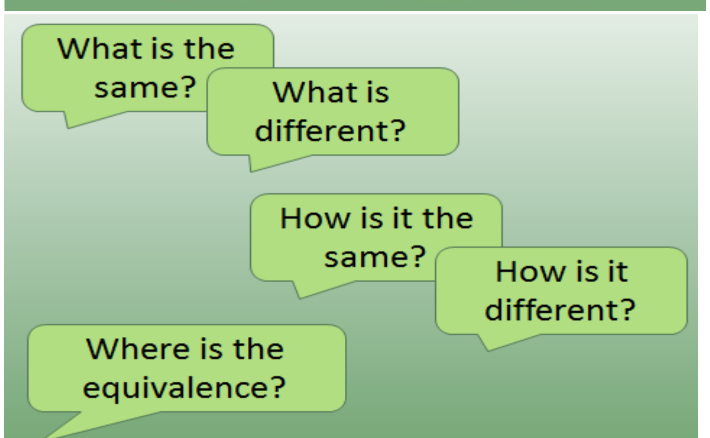
bar models

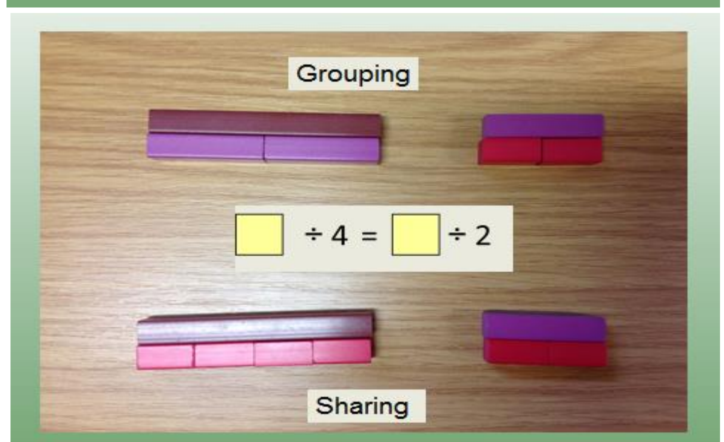

What early experiences begin to develop children's ideas of equivalence ?
I'm nearly as tall as you!
It's not fair! You've had it longer than me!


Exploring equivalence with balance scales
I've put some pegs on the 8 hook and some on the 6 hook. How many did I put on each?
10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10
If I put 3 pegs on the 8 hook on the left hand side, how else could I use up to 4 pegs to make it balance?
• Add the weights: http://nrich.maths.org/4763
• Getting the balance: http://nrich.maths.org/5676
• Balance of halves http://nrich.maths.org/5677


Equivalence – how is it the same?
One-to-one matching
The property that makes these sets equivalent is their 'threeness'.


Explore…
6 x ☐ = 2 x ☐
☐ ÷ 4 = ☐ ÷ 2
How can representations support your thinking?


What is the same?
What is different?
How is it the same?
How is it different?
Where is the equivalence?


Grouping
☐ ÷ 4 = ☐ ÷ 2
Sharing

bar models